International Journal of Recent Research and Review, Vol. IX, Issue 2, June 2016
ISSN 2277 – 8322

Stimulation of Stable Isotope Beta-Decay by Powerful Heating of Substance and *p*-Nucleus Synthesis in Massive Stars

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Abstract **- We have demonstrated that the strong heating of the matter stimulates the beta-decay of isotopes that are stable under normal conditions. In the medium heated to the "nuclear"temperature of 200–300 keV we calculate half-lives of 33 stable nucleiin the range of mass numbers from 74 to 196. We use the physical mechanisms of photo-beta-decay and thermal beta-transitions. They also become the basis of a new model of the synthesis process of *p*-isotopes at the quasi-equilibrium stage of a massive star evolution. Performing the calculation of the abundances,it is necessary to solve a set of kinetic equations for beta-decay chains leading to *p*-nucleus and consisting of two stable even-even nuclei and intermediate multi-beta-decay odd-odd nucleus. In these calculations for the first time we simultaneously take into account all the types of thermal beta-transitions (electron and positron beta-transitions, *K*-capture) and nuclear photo-beta-decay. We investigated the stage of high-temperature burning of an oxygen layer in a massive star. It is found that during burning for 20 of 33 *p*-nuclei it is possible to get their observed "solar" concentrations.**

Keywords **- p-nucleus, abundance, β-transition, supernova**

I. INTRODUCTION

Beta processes rarely play a major role in the synthesis of chemical elements because of their slowness although at quite long quasi-equilibrium stages of evolution of massive stars they can compete with other processes such as the capture of neutrons by nuclei. An example would be the *s*-process in which beta-decay is responsible for the synthesis of many stable isotopes along with the slow neutron capture [1, 2]. If we consider the explosive fusion of nuclei that now play the key role in solving the problem of the origin of most chemical elements especially behind the iron maximum, then beta-processes are not critical in it,as a rule, although they are considered in the solution of systems of kinetic equations.

The model of explosive synthesisis also considered to be promising for solving the known problem of the originof poor-neutron isotopes or *p*-nuclei [1-10]. As it is well known,in the *s*-process these isotopes cannot be obtained, so they are called bypassed. The fact is that after the captureof a neutron,there is a chain of successive beta-decays which end with a stable even-even (A, Z) nucleus where A and Z are mass and charge numbers, respectively. The following beta-transition, (A, Z)→(A, Z+1),where the (A, Z+1) nucleus is odd-odd, is blocked because of the energy threshold. If we overcome it, we would come again to the even-even *p*-isotope, (A, Z+2), and the problem would be solved. Therefore, it is necessary to offer a physical mechanism for overcoming this energy threshold. Such attempts have been undertaken repeatedly by using neutrino radiation [11],collisional beta-decay [12], thermal beta-decay [13] and photo-beta-decay [14,15,16,17]. However, except in specific cases, it was possible to obtain only aqualitative agreement between the calculated and experimental abundances.

In the models based on explosive synthesis of chemical elements there is no problem of overcoming of energy threshold because in supernova explosionsa shock wave propagates in the stellar matter. Therefore protons, alpha particles, light nuclei which are already available or produced in nuclear reactions under its influence can overcome the Coulomb barrier and can be absorbed directly by the (A, Z) nucleus. As a result of such processes, *p*-isotopes can appear. As it was demonstrated in the results of research in this area (see, e.g., reviews [2] and [6]), the calculated and observed

abundances differ already by two or three times in the average, not by orders of magnitude, and this result is encouraging. However, as it is notedin [9], there are still some «problematic» *p*-isotopes even in this approach. It is possible to obtain their abundance by changing the rates of some key reactions of *s*-process, and thus obtaining aproblem already with the *s*-nuclei.

Despite the success of the models considering the synthesis of *p*-nuclei at supernova explosions, the paper will offer a model of the synthesis process suitable for quasi-equilibrium stages in the evolution of massive stars, as well as for pre-supernova stage. The fact is that up to now in the problem of origin of chemical *p*-elements all effects accelerating beta-processes in a highly heated matter have not been considered in one and the same study. As it was demonstrated in [18], the observed abundances of some "problematic" *p*-nuclei can be obtained even during relatively cold helium layer burning in a massive star just in a complex approach. It involves the simultaneous consideration of all beta-processes types in a highly heated matter: thermal electron capture,electron and positron beta-decays and photo-beta-decay. If the temperature of the matter has anuclear scale (in energy unitsit is not less than 200–300 keV), significant acceleration of beta-transitions is possible, especially for those inhibited by the quantum selection rules. Moreover, the beta-decay is stimulated, even for the isotopes that are beta-stable under normal conditions. At the same time, high-temperature field can significantly change not only the rates of electron and positron beta-transitions, and even their ownratio, as it is shown in the example of multi-beta-decay nuclei [19].

The aim of the present paper is to demonstrate that not all possible solutionsof the *p*-isotopes origin problem have been exhausted within the model of quasi-equilibrium stages of stellar evolution by using integrated approach. It will be shown that at the stage of burning the oxygen layer of the massive star, thehigh-temperature field allows to overcome the energy threshold that prohibits beta-transition, $(A, Z)\rightarrow(A, Z+1)$,under normal conditions. As it turned out, there is enough time for this stage to get the required intensity of the thermal electron beta-decay and photo-beta-decay of the stable even-even (A, Z) nucleus. Its decay creates the possibility of realizing a chain of beta-decays, $(A, Z)\rightarrow(A, Z+1)\rightarrow(A, Z+2)$, resulting in the *p*-nucleus, $(A, Z+2)$. A *p*-nucleus synthesis model using a set of kinetic equations for the chain, $(A, Z)\rightarrow(A, Z+1)\rightarrow(A, Z+2)$, will be also proposed. This model takes into account all types of thermal beta- and photo-beta-transitions at all stages stimultaneously.

Table I

Dependence of beta-stable nuclei half-lives, $t_{1/2}(T)$, on temperature, T, inextremely heated medium. Experimental data on the threshold energies are taken from [20].

Progenitor nucleus	Threshold energy (keV)	half-life,$t_{1/2}(T)$		Progenitor nucleus	Threshold energy (keV)	half-life,$t_{1/2}(T)$	
		2×10^9 K	3×10^9 K			2×10^9 K	3×10^9 K
^{74}Ge	2562	3.4×10^3y	13 y	^{130}Xe	2983	2.2×10^4 y	47.6y
^{78}Se	3574	6.5×10^5 y	370 y	^{132}Xe	2120	279 y	111 d
^{80}Se	1871	9.5 y	29 d	^{136}Ba	2870	2.0×10^3y	5.4 y
^{84}Kr	2681	2.5×10^3y	4.8 y	^{138}Ba	1737	285 d	4.1 d
^{92}Zr	2006	2.3×10^3y	15.2 y	^{144}Nd	2332	2.1×10^3 y	10.5 y
^{94}Zr	902	10.9 d	16.1 h	^{152}Sm	1818	31.7 y	207 d
^{96}Mo	2973	9.8×10^3 y	22.2 y	^{156}Gd	2444	794 y	4.4 y
^{98}Mo	1684	1.5 y	98.3 d	^{158}Gd	1220	36 d	3.5 d
^{102}Ru	2323	152 y	240 d	^{162}Dy	2140	317 y	2.8 y
^{106}Pd	2965	1.4 d	33.3 m	^{164}Dy	987	33 d	3.0 d
^{108}Pd	1918	44.4 y	309 d	^{168}Er	1679	34.9 y	321 d

^{110}Pd	893	12.7 d	1.5 d	^{174}Yb	1374	1.3 y	30 d
^{112}Cd	2586	2.6x10^{3}y	12.1 y	^{180}Hf	854	3.9 d	12.8 h
^{114}Cd	1453	228 d	7.9 d	^{184}W	1483	170 y	1.5 y
^{120}Sn	978	6.0x10^{3}y	16.5 y	^{190}Os	2000	330 y	3.4 y
^{124}Te	3160	3.5x10^{5}y	146y	^{196}Pt	1506	112 y	1.9 y
^{126}Te	2155	63.5 y	230 d				

II .BETA-DECAY OF STABLE ISOTOPES IN EXTREMELY HEATED MATTER

Keeping in mind the problem of the chemical origin of *p*-elements, we will consider a stable even-even (*A, Z*) nuclei, the beta-decay of which in a high temperature field can contribute to the accumulation of the stable *p*-nuclei, (*A, Z* +2), in the matter by the chain of beta-transitions, (*A*, *Z*)→(*A*, *Z*+1)→(*A*, *Z*+2), as well. There are such 33 nuclei with the mass number range from 74 to 196; they are all listed in Table I. The values of energy thresholds for beta-transitions, (*A*, *Z*)→(*A*, *Z*+1), are also shown here.

In the extremely heated medium the energetically forbidden electron beta-transition, (*A*, *Z*)→(*A*, *Z*+1), becomes possible due to the mechanisms of thermal and photo-beta-transitions. In the first case, in the (*A, Z*) nucleus the medium heating populates the excited states, the energies of which are higher than the threshold. This opens the beta-decay channel if there are no obstacles due to the quantum selection rules. In the second case we mean an endothermic beta-decay for which the necessary energy to overcome the threshold is taken from the electromagnetic field with the Planck spectrum of frequencies. Such decay is possible both from the ground and from the populated excited states with the energies below the threshold, but again if there are no obstacles for beta-decay due to the quantum selection rules. Below,following the paper [18], we present expressions for the rates of thermal beta-decays and electron photo-beta-decay.

In the medium heated up to the temperature, *T*, the total rate, $\lambda_{tot}^{(\xi)}((A,Z);T)$, of the electron beta-decay ($\xi = \beta^-$) or of the photo-beta-decay ($\xi = \gamma\beta^-$) of the (*A, Z*) nucleus has the form,

$$\ddot{e}_{tot}^{(\hat{i})}\left((A,Z_i);T\right) = \sum_{a,b} P(E_a,T)\,\ddot{e}_{a\to b}^{(\hat{i})}\left((A,Z_i);\ddot{A}_{ab}^{(\hat{i})}\right) \quad ..(1)$$

Here, $P(E_a,T)$ isthe population probability of the *a*th state of the (*A, Z*) nucleus with the energy, E_a, measured with respect to the ground state,

$$P(E_a,T) = \frac{2j_a+1}{G(T)}\exp(-E_a/kT), \quad ..(2)$$

Where j_a is the total spin of the *a*th state of the (*A, Z*) nucleus, *k* is theBoltzmann constant, and *G (T)* is the statistical sum,

$$G(T) = \sum_a (2j_a+1)\exp(-E_a/kT) \quad ..(3)$$

$\lambda_{a\to b}^{(\xi)}((A,Z);\Delta_{ab}^{(\xi)})$ is the partial transition rate from the *a*th state of the parent nucleus, (*A, Z*), to the *b*th state of the daughter nucleus, (*A, Z* +1), with energy, E_b, measured with respect to the ground state energy, $\Delta_{ab}^{(\xi)}$ is the transition energy.

Later only allowed beta-transitions will be selected as the most intense ones (as the analysis of the level schemes of isotopes under consideration shows, it is always possible tofulfill the conditions). In this case the partial rate of beta-transition can be calculated by the following formula(we use here after the"natural" system of units, $\hbar = c = m_e = 1$,where *c* is the speed of light and m_e is theelectron mass),

$$\ddot{e}_{a\to b}^{(\tilde{a}\hat{a}^-)}\left((A,Z);\ddot{A}_{ab}^{(\hat{a}^-)}\right) = (2\eth^3)^{-1}\int_1^{\ddot{A}_{ab}^{(\hat{a}^-)}} E(E^2-1)^{\frac{1}{2}}\left(\ddot{A}_{ab}^{(\hat{a}^-)}-E\right)^2 C_{ab}^{(0)}(Z+1,E)F_0(Z+1,E)dE \quad ..(4)$$

where, $\Delta_{ab}^{(\beta^-)} = Q_\beta + E_a - E_b$, $Q_\beta = M(A,Z) - M(A,Z+1)$ is the standard notation for the energy released in beta-transition between the ground states of the parent and daughter nucleiin the theory of beta-decay, *M (A, Z)* is the atomic mass of the (*A, Z*) nucleus, $F_0(Z,E)$ is the Coulomb Fermi function (there is an analytical expression for it and tables of its values, e.g., in [21]) and $C_{ab}^{(0)}(Z+1,E)$ is the form of the spectrum for

the allowed beta-transition, $a \rightarrow b$. With sufficient accuracy it can be calculated by the formula,

$$C_{ab}^{(0)}(Z+1,E) \approx \left|M_V^{(a\to b)}\right|^2 + \left|M_A^{(a\to b)}\right|^2 \quad ,.(5)$$

Where $M_V^{(a\to b)} = g_V \int (1)_{ab}$ and $M_A^{(a\to b)} = g_A \int (\boldsymbol{\sigma})_{ab}$ are the Fermi and Gamow-Teller nuclear matrix elements for the beta-transition, $a \rightarrow b$, (in the standard notation), g_V and g_A are corresponding coupling constants.

Similar to [17,18,19], we also have to deal with the calculation of nuclea rmatrix elements for the beta-transitions between the excited states of the even-even parent (A, Z) nucleus and the odd-odd daughter (*A, Z* +1) nucleus. Because of the complexity of the structure of these states, a large number of beta-transition and the difficulties with the choice of appropriate nuclear models, we prefer to use the average values of the nuclear matrix elements as before. They can be obtained by focusing on the typical values of $\lg f_0 t_{1/2}$ = 4.0-5.5 for unfavored allowed transitions. For them, as itis well known ,$f_0 t_{1/2}$ is directly related to the values of the matrix elements by the formula,

$$f_0 t_{1/2} = 2\pi^3 \ln 2 / \left(\left|M_V^{(a\to b)}\right|^2 + \left|M_A^{(a\to b)}\right|^2\right)$$

It allowsto takethe average values of the matrix elements in calculation of the form factor, $C_{ab}^{(0)}(Z+1,E)$ (see equation (5)), i.e., to assume

$$C_{ab}^{(0)}(Z+1,E) \approx \left\langle \left|M_V^{(a\to b)}\right|^2 + \left|M_A^{(a\to b)}\right|^2 \right\rangle = 2\eth^3 ln2/\langle f_0 t_{1/2}\rangle \quad ..(6)$$

Here, $\langle f_0 t_{1/2}\rangle$ is the average value of the reduced life time of the unfavored allowed beta-transition which must be defined by selecting a specific value from the range of $10^{4.0} - 10^{5.0}$s. Substituting (6) into (4), for the partial rate of beta-transition,we finally get,

$$\ddot{e}_{a\to b}^{(\hat{a}^-)}\left((A,Z_i);\ddot{A}_{ab}^{(\hat{a}^-)}\right) = \frac{In\,2}{\langle f_0 t_{1/2}\rangle}\int_1^{\ddot{A}_{ab}^{(\hat{a}^-)}} E(E^2-1)^{1/2}(\ddot{A}_{ab}^{(\hat{a}^-)} - E)^2 F_0(Z_i+1,E)dE \quad ..(7)$$

Following the papers [14, 16, 18], the expression for the partial rate of the allowed photo-beta-decay can be written as

$$\ddot{e}_{a\to b}^{(\tilde{a}\hat{a}^-)}\left((A,Z_i);\ddot{A}_{ab}^{(\hat{a}^-)}\right) = \frac{\acute{a}In\,2}{\eth\langle f_0 t_{1/2}\rangle}\int_{\ddot{A}_{ab}^{(\tilde{a}\hat{a}^-)}}^{\infty}\frac{d\grave{u}}{\grave{u}}\frac{G(\grave{u},\ddot{A}_{ab}^{(\tilde{a}\hat{a}^-)})}{e^{(\grave{u}/kT)}-1} \quad ..(8)$$

where α= 1/137.04 is the fine structure constant and $\Delta_{ab}^{(\gamma\beta^-)} = Q_\beta + E_a - E_b$ is the energy threshold for photo-beta-transition. Now, unlike the β-decay, Q_β<0, and endothermic photo-beta decay is possible only under the condition,$\omega > \Delta_{ab}^{(\gamma\beta^-)}$. At last,

$$G(\omega,\Delta_{ab}^{(\gamma\beta^-)}) = \int_1^{\omega-\Delta_{ab}^{(\gamma\beta^-)}+1} (\omega-E-\Delta_{ab}^{(\gamma\beta^-)}+1)^2\{2(\omega-E)(E^2-1)^{1/2} + (\omega^2-2\omega E+2E^2)\ln[E+(E^2-1)^{1/2}]\}F_0(Z_i+1,E)dE \quad ..(9)$$

Let us introduce the total rate of the electron beta-decay of the (*A, Z*) nucleus,

$$\lambda_{tot}(T) \equiv \lambda_1 = \lambda_{tot}^{(\beta^-)}((A,Z);T) + \lambda_{tot}^{(\gamma\beta^-)}((A,Z);T) \quad ..(10)$$

Then the half-life, $t_{1/2}(T)$,of the stable (*A, Z*) nucleus in the high temperature field is

$$t_{1/2}(T) = \ln 2 / \lambda_{tot}(T) \quad ..(11)$$

The numerical results of the half-life values as the functions of the temperature are presented in the same Table I. The given values have maximum values since they were calculated with the maximum values of the nuclear matrix elements. They correspond to the value of $\langle f_0 t_{1/2}\rangle = 10^4$s. Taking into account the complexity of excited nuclear states of even-even mother and odd-odd daughter nuclei, $\langle f_0 t_{1/2}\rangle \approx 3.2\text{x}10^5$s would be probably a close-to-real value which corresponds to $\lg f_0 t_{1/2}$=5.5. Then accordingly all the half-lives in Table 1 should be increased by the factor of 32. As we can see from Table 1, the values of threshold energy and half-life, $t_{1/2}(T)$, correlate upon the average: the higher the threshold energy, the greater the half-life. However, this dependence is not directly proportional. It can be corrected by the number of allowed beta-transitions in the selected energy ranges, i.e. it can depend on the quantum characteristics of nuclear states.

III. MODEL OF *p*-ISOTOPE SYNTHESIS PROCESS IN BETA-DECAY CHANNELS

Let us consider the chain ofthe beta-decays,$(A,Z)\leftrightarrow(A,Z+1)\rightarrow(A,Z+2)$. The final (*A*,*Z*+2)

nucleus in itis an even-even*p*-nucleus, and the initial (*A*,*Z*) nucleus is a beta-stable even-even one which, as it was demonstrated in the previous section,becomes beta-active in a heated medium. As a rule, the intermediate (*A*,*Z*+1) nucleus is amulti-beta-decay state. It means that its positron beta-decay and electron capture take place implementing the beta-transition, (*A*,*Z*+1)→(*A*,*Z*) and also the beta-decay, (*A*,*Z*+1)→(*A*,*Z*+2). In a few cases, under terrestrial conditions, the fraction of beta-decay may be small but in extremely heated medium it significantly increases [19].

Let us formulate the foundation of the proposed *p*-nucleus synthesis model:

1. The triad of the nuclei, (*A*,*Z*), (*A*,*Z*+1) and (*A*,*Z*+2), is considered as isolated.
2. In the chain ,(*A,Z*)↔(*A,Z*+1)→(*A,Z*+2), only beta-processes of all types are taken into account. These processes can be stimulated, as in the case of the stable (*A*, *Z*) nucleus, or enhanced, as in other cases, by thehigh-temperature field of the medium.
3. For the calculation of the (*A*, *Z*), (*A*, *Z*+1) and (*A*, *Z*+2) nucleus abundances a set of kinetic equations is to besolved.
4. The"solar" abundances of the stable even-even (*A*, *Z*) nuclei are taken as the initial data and the "solar" abundances of the even-even (*A*, *Z*+2) *p*-nuclei are considered as the reference points.
5. It is assumed thatthe temperature of the medium is not changed during the whole synthesis of nuclides.

The main purpose of the calculations is to show that using the comprehensive approach is possible to accumulate *p*-nuclei in significant quantities at the quasi-equilibrium stages of the evolution of massive stars with arelatively high temperature of the matter. As in the previous section, in the medium heated up to the temperature, *T*, we will calculate the total rate of all types of the beta-decay considering beta-transitions and photo-beta-transitions from the excited nuclear states. Again we select only the allowed beta-transitions (in all cases this condition is feasible).

The calculation of the total rate of the (*A*, *Z*+1) nucleus beta-decay, as in the case of the same decay of the (*A*, *Z*) nucleus,can be made by formulas (1) and (7)-(10), where *Z* should be replaced by *Z*+1. To find the total rate of the thermal positron beta-decay,of the (*A*, *Z*+1) nucleus, we can also use the formula (1) and (7). In formula (1) we have to put $\xi = \beta^+$ and replace *Z* by *Z*+1. In formula (7), in the calculation of the partialrate, $\lambda_{a\to b}^{(\beta^+)}((A, Z+1); \Delta_{ab}^{(\beta^+)})$, we must replace *Z* by *Z* +1, and $\Delta_{ab}^{(\beta^-)}$ by $\Delta_{ab}^{(\beta^+)} = Q_\beta^{(+)} + E_a - E_b$, where $Q_\beta^{(+)}$ is the energy released in the positron beta-transition between the ground states of the (A, Z +1) and (A, Z) nuclei, $Q_\beta^{(+)} = M(A, Z+1) - M(A, Z) - 2$ (the positron beta-transition will occuronly if $\Delta_{ab}^{(\beta^+)} > 0$). Furthermore, we must use the Coulomb Fermi function for the positron beta-decay. In this case *Z* must be used instead of *Z* +1 in itsargument.

Electron capture can also contribute to the transition, (*A*, *Z* +1)→(*A*, *Z*), along with the positron beta-decay. We will consider the most intense allowed electron *K*-capture. The total rate of the *K*-capture in the heated medium, $\lambda_{tot}^{(\varepsilon_K)}((A, Z+1); T)$,can again be calculated by using formula (1), by putting $\xi=\varepsilon_K$ in it and by replacing *Z* by *Z*+1. For the calculation of the partialrate, $\lambda_{a\to b}^{(\varepsilon_K)}((A, Z+1); \Delta_{ab}^{(\varepsilon_K)})$, used in our approach to estimate the values of the nuclear matrix elements,we can use the expression,

$$\lambda_{a\to b}^{(\varepsilon_K)}((A, Z+1); \Delta_{ab}^{(\varepsilon_K)}) \approx \frac{\pi \ln 2}{2\langle f_0 t_{1/2}\rangle} q_K^2 \beta_K^2. \qquad ..(12)$$

Here, $q_K \equiv \Delta_{ab}^{(\varepsilon_K)} = Q_\beta^{(+)} + 1 - |E_K| + E_a - E_b$, where $Q_\beta^{(+)}$ is defined above, and E_K and β_K are the energy and the Coulomb amplitude wave function of the bound *K*-electron, respectively (there is a table of values in [21] for them). In (12) the factor taking into account the electron exchange is omitted, as for the *K*-shell, its value is almost equal to the unity.

In highly heated medium,as noted in [22], electron capture can be highly suppressed due to the almost complete ionization of the atoms of the matter.

At thesame time, using the specific calculations of the atomic level populations,it is necessary to know the density of the electron gas and the corresponding isotopes in stellar matter. These values depend on the star model and there can also be problems.To avoid this difficulty, we also assume the *K*-shells to be totally populated in atoms with (*A*, *Z*+1) nuclei, especially since positron beta-decays,as a rule, have intensities comparable in magnitude. This approach is approved by the fact that the main purpose of calculations is not to give exact values of abundances but to demonstrate the principal possibility of *p*-isotope accumulation in sufficient quantities at the quasi-equilibrium stages of the star evolution. Neglecting electron *K*-capture will only increase the total values of *p*-nucleus abundancesdue to the relative increase in the fraction of beta-decay in the multi-beta-decay of (*A*, *Z*+1) nuclei.

Let us introduce, along with the total rate, λ_1, of the (*A*, *Z*) nucleus beta-decay (equation (10)), the total rate of beta-transition, (*A*, *Z*+1)→(*A*, *Z*),

$$\ddot{e}_{tot}^{(+)}(T) \equiv \ddot{e}_2 = \ddot{e}_{tot}^{(\hat{a}^+)}\big((A,Z+1);T\big) + \ddot{e}_{tot}^{(\mathring{a}_k)}((A,Z+1);T) \quad ..(13)$$

and the total rate of beta-transition, (*A*, *Z*+1)→(*A*, *Z*+2),

$$\ddot{e}_{tot}^{(-)}(T) \equiv \ddot{e}_3 = \ddot{e}_{tot}^{(\hat{a}^-)}\big((A,Z+1);T\big) + \ddot{e}_{tot}^{(\tilde{a}\hat{a}^-)}((A,Z+1);T) \quad ..(14)$$

Let us assume that at the initial time,t = 0,the abundance, $N((A,Z);t) \equiv N_1(t)$,of stable (*A*, *Z*) nuclei is equal to N_0, and the abundances, $N((A,Z+1);t) \equiv N_2(t)$ and $N((A,Z+2);t) \equiv N_3(t)$, of (*A*, *Z*+1) and (*A*, *Z*+2) nuclei, respectively, are equal to zero.They are the initial conditions for the set of kinetic equations describing the accumulation of (*A*, *Z*+2) *p*-nuclei at the moment of time,*t*,

$$\frac{dN_1}{dt} = -\lambda_1 N_1(t) + \lambda_2 N_2(t);$$
$$\frac{dN_2}{dt} = \lambda_1 N_1(t) - (\lambda_2 + \lambda_3) N_2(t); \quad ..(15)$$
$$\frac{dN_3}{dt} = \lambda_3 N_2(t).$$

This set of equations can be solved numerically, and we can also obtain an analytical solution. Finally, the value of $N_3(t)$ is of interest.It has the form,

$$N_3(t) = N_0\left\{1 - \frac{1}{2}\left[e^{-\frac{\ddot{a}+t}{2}} + e^{-\frac{\ddot{a}-t}{2}}\right] - \frac{\ddot{e}_{123}}{\ddot{a}}\sinh(\ddot{a}t/2)e^{-\frac{\ddot{e}_{123}t}{2}}\right\} \quad ..(16)$$

Here, we introduced the notations,

$$\delta = (\lambda_{123}^2 - 4\lambda_1\lambda_3)^{1/2}; \delta_{\pm} = \lambda_{123} \pm \delta; \lambda_{123} = \lambda_1 + \lambda_2 + \lambda_3.$$

We can consider the particular case of (16) assuming fulfillment of inequalities, $\lambda_1 << \lambda_2$, λ_3 and $\lambda_1\tau << 1$ (τ is the time of the stage duration). Then

$$\delta \approx \lambda_{123} - 2\lambda_1\lambda_3/\lambda_{123}, \delta_+ \approx 2(\lambda_{123} - \lambda_1\lambda_3/\lambda_{123}), \delta_- \approx 2\lambda_1\lambda_3/\lambda_{123}$$

and $\lambda_{123} \approx \lambda_2 + \lambda_3$.

For most beta-decay chains, (*A*,*Z*)↔(*A*,*Z*+1)→(*A*,*Z*+2), these conditions hold. Then from equation(16) we obtain

$$N_3(\tau) \approx N_0\lambda_1\tau K \quad ..(17)$$

where $K = \lambda_3/(\lambda_2 + \lambda_3)$ is the branching coefficientor the fraction of electron beta-decay in the total decay rate of the (*A*, *Z*+1) nucleus. The approximate formula (17) was previously usedin the models for studying processes of the *p*-nucleus synthesis at the stage of the oxygen burningin the stars, bothin the thermal beta-decay [13] and photo-beta-decay [16, 17].

IV. NUMERICAL RESULTS AND DISCUSSION

The numerical results for the abundances of all known *p*-nuclei obtained within the proposed scheme are given in Table 2. We considered a relatively hot stage of the oxygen burning during the evolution of a massive star. We chose the stage duration equal to 5 months. In this case the temperature of the matter reaches (2–3) T_9 (hereafter we use the notation,T_n = 10^n K) that correspondsto the "nuclear" temperature of about 200-300 keV in energy units. Experimental data of the isotope characteristics were taken from [20].We chose the energy range of the nucleus excited states in such a way that there were several allowed beta-transitions in them. Corresponding triads are given in Table II. As an example, there are parts of the isotope level schemes from the triad,^{164}Dy, ^{164}Ho and ^{164}Er,

given in Figure 1. The arrows indicate only electron thermal beta and photo-beta-transitions, $^{164}Dy \rightarrow ^{164}Ho$ and $^{164}Ho \rightarrow ^{164}Er$, not to overburden the figure. These transitions were taken into account in the calculation of the total beta-decay rates.

The analysis of the data from Table2 shows that for 20 *p*-isotopes from 33, their observed abundances may be very well produced during burning of the oxygen layer. These are the isotopes, ^{74}Se, ^{80}Kr, ^{84}Sr, ^{94}Mo, ^{98}Ru, ^{102}Pd, $^{106,108,110}Cd$, ^{114}Sn, ^{120}Te, ^{126}Xe, ^{132}Ba, ^{138}Se, ^{152}Gd, ^{158}Dy, ^{164}Er, ^{168}Yb, ^{174}Hf and ^{180}W. For the remaining 13 nuclides, apparently, beta-decay mechanism does not play asignificant role in the processes of their synthesis. As a matter of fact, the latter conclusion should still be examined by investigating the role of electron *K*-capture which is suppressed in a highly heated matter. The present calculations do not take it into account and therefore the value of the branching factor, *K*, may be significantly underestimated.

In some cases the calculated abundances significantly exceed the observed ones. In our opinion, it is not important. The fact is that when calculating the beta-decay rates as well as the half-lives of nuclei in the previous section, them aximum estimationin magnitude of the nuclear matrix elements corresponding to the value of $\lg f_0 t_{1/2} = 4.0$ was used. If we take as a basis, perhaps, a more appropriate quantity, 5.5, taking into account the complexity of the nuclear states, then the theoretical abundances given in Table 2 should be divided by 32, or choosing the value $\lg f_0 t_{1/2}$ in the interval of 4.0-5.5, by the factor from 1 to 32. It is important that in the calculations the medium temperature keeps constant throughout the stage whereas, for example, its reduction strongly affects the rates of beta-processes (cf. columns 1 and 2 in Tables 1 and 2). It is necessary to add that the calculations are focused on the currently observed abundances of progenitor (*A*, *Z*) nuclei and (*A*, *Z*+2) *p*-nuclei which could be different at the stage of the oxygen layer burning.

A certain amount of *p*-isotopes can further be synthesized in a similar way at the next stage of a massive star evolution, at the stage of silicon burning. Although it is relatively short, only one day, the stellar matter can be heated up even more, up to the temperature of $5T_9$, or about 0.5 MeV in energy units. To estimate how much the abundances of *p*-nuclei obtained in the previous stage using the same pattern would increase the yield of two isotopes, ^{74}Se and ^{78}Kr, was calculated under the assumption that the temperature of the matter has its maximum value, i.e. $5T_9$, and also keeps the same for all the stages. It is also taken into account that at the previous stage the loss of progenitor nuclei, ^{74}Ge and ^{78}Se, is small and we can take the same initial values of their abundance, N_0, as in Table2. As a result, by usingthe maximum values of the nuclear matrix elements $N_3(^{74}Se)=0.633$ and $N_3(^{78}Kr)=6.8\text{x}10^{-4}$ are obtained. The first result is comparable in value with the abundance shown in Table II for the temperature equal to$3T_9$, and the latter is more than 4 times higher. However,in the second case this abundance will be muchless than the observed one. Although the stage length for the silicon burningis much smaller than for the oxygen burning, the medium temperature increasing from $3T_9$ to $5T_9$ results in a dramatic increase in the total rate of beta-transition, $^{74}Ge \rightarrow ^{74}As$ and $^{78}Se \rightarrow ^{78}Br$, in the first case by two orders and in the second case nearly by three orders.Then the total rates of the remaining beta

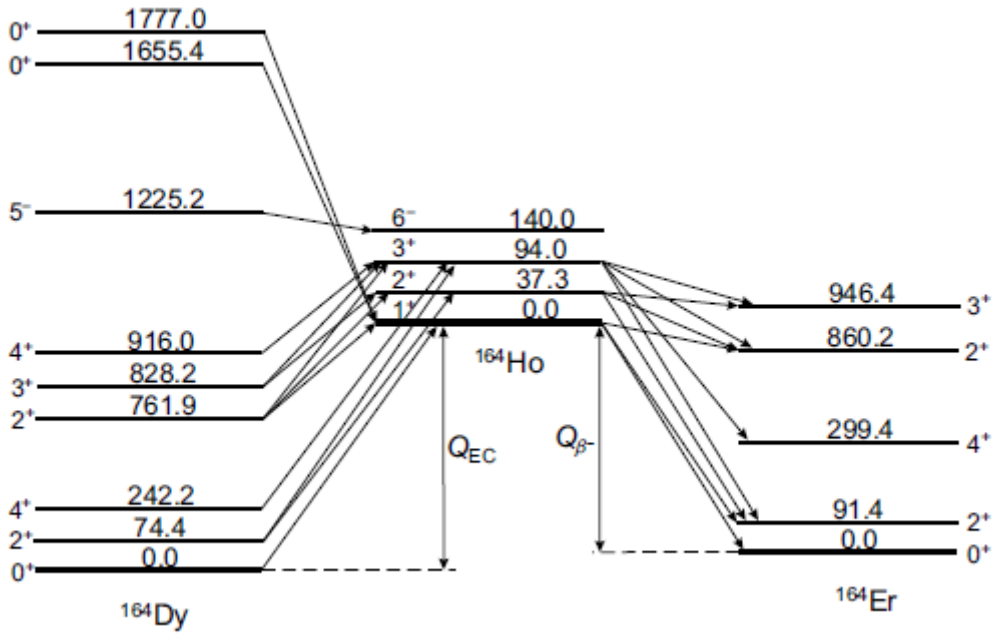

Fig. 1 Parts of level schemes (in keV) of isotopes from the triad, ^{164}Dy, ^{164}Ho and ^{164}Er (data are taken from[20]). The arrows indicate the allowed electron beta and photo-beta –transitions taken into account in the calculation of the totalrate. Q_{EC} = 986.7keV, $Q_{\beta-}$= 962.5 keV

--transitions are changed smoothly and not so much, and the dramatic increase in the beta-transition rates at the stageof mounting the energy threshold leads to a significant result.

V. CONCLUSION

In our view, the main conclusion is that the synthesis of most *p*-isotopes could easily occur at quasi-equilibrium stages of massive star evolution with heating the matter up to the "nuclear" temperatures. It could be the stages of the oxygen and silicon burning although the latter statement requires a further study.The fact that earlier the approaches to the problem of *p*-nuclei synthesis in beta-decay channels did not give meaningful and quantitative results, in our opinion it is explained by the lack of a comprehensive approach. Exactly it is the consideration of all types of beta-decay in the whole chain of transitions, $(A,Z)\leftrightarrow(A,Z+1)\rightarrow(A,Z+2)$, starting from the stable nucleus and full accounting of their intensification in extremely heated stellar matter, that makes it possible to obtain the observed abundances for most of the *p*-isotopes.

In any case, the chain of kinetic equations solved in the standard theory of nucleosynthesis in the calculation of the abundances of *s*-nuclei which are the initial nucleus (*A, Z*) in our model could be extended to the (*A, Z*+2) *p*-nucleus. Then in some cases it would not need to pick up most of *p*-isotopes in a separate category of "bypassed".

The calculation of *p*-nucleus abundances according to the proposedor similar model considering quasi-equilibrium stages of the massive star evolution with strong heating ofthe matter can be useful for modeling of explosive nucleosynthesis. Then it is possible to use the calculated abundances as the initial data for the sets of kinetic equations being solved in such models.

Table II

Calculated values of *p*-nucleus abundances . Parameter values are used: τ = 5 months, $\langle f_0 t_{1/2} \rangle_0 = 10^4$ s.

Experimental data ("solar"abundances) are taken from [23] (they are normalized to $N(Si)=10^6$).

Progenitor nucleus (A, Z)	Parent Nucleus, $(A, Z+1)$ (multi-decay)	*p*-nucleus $(A, Z+2)$	"Solar" abundance of progenitor nucleus (A, Z) Exp.	"Solar" Abundance of $N(A, Z+2)$ *p*-nucleus Exp.	Abundance of *p*-nucleus $N(A, Z+2)$ Theor.	
					$2\text{x}10^9$ K	$3\text{x}10^9$ K
$^{74}_{32}Ge$	$^{74}_{33}As$	$^{74}_{34}Se$	42.8	0.580	$2.5\cdot10^{-3}$	0.705
$^{78}_{34}Se$	$^{78}_{35}Br$	$^{78}_{36}Kr$	15.8	0.146	$1.1\cdot10^{-7}$	$1.7\cdot10^{-4}$
$^{80}_{34}Se$	$^{80}_{35}Br$	$^{80}_{36}Kr$	33.4	0.940	0.970	30.6
$^{84}_{36}Kr$	$^{84}_{37}Rb$	$^{84}_{38}Sr$	23.5	0.128	$5.6\cdot10^{-4}$	0.523
$^{92}_{40}Zr$	$^{92}_{41}Nb$	$^{92}_{42}Mo$	2.05	0.634	$4.5\cdot10^{-6}$	$3.2\cdot10^{-3}$
$^{94}_{40}Zr$	$^{94}_{41}Nb$	$^{94}_{42}Mo$	2.09	0.361	2.08	2.06
$^{96}_{42}Mo$	$^{96}_{43}Tc$	$^{96}_{44}Ru$	0.661	0.105	$1.4\cdot10^{-10}$	$5.1\cdot10^{-7}$
$^{98}_{42}Mo$	$^{98}_{43}Tc$	$^{98}_{44}Ru$	0.951	0.036	0.109	0.729
$^{102}_{44}Ru$	$^{102}_{45}Rh$	$^{102}_{46}Pd$	0.601	0.013	$1.5\cdot10^{-4}$	0.025
$^{106}_{46}Pd$	$^{106}_{47}Ag$	$^{106}_{48}Cd$	0.355	0.019	0.060	0.355
$^{108}_{46}Pd$	$^{108}_{47}Ag$	$^{108}_{48}Cd$	0.347	0.014	$1.6\cdot10^{-3}$	0.071
$^{110}_{46}Pd$	$^{110}_{47}Ag$	$^{110}_{48}Cd$	0.154	0.020	0.153	0.153
$^{112}_{48}Cd$	$^{112}_{49}In$	$^{112}_{50}Sn$	0.373	0.036	$7.4\cdot10^{-6}$	$1.7\cdot10^{-3}$
$^{114}_{48}Cd$	$^{114}_{49}In$	$^{114}_{50}Sn$	0.447	0.024	0.143	0.368
$^{120}_{50}Sn$	$^{120}_{51}Sb$	$^{120}_{52}Te$	1.22	$5.8\cdot10^{-3}$	$9.1\cdot10^{-6}$	$3.6\cdot10^{-3}$
$^{124}_{52}Te$	$^{124}_{53}I$	$^{124}_{54}Xe$	0.299	$7.4\cdot10^{-3}$	$1.1\cdot10^{-10}$	$4.5\cdot10^{-7}$
$^{126}_{52}Te$	$^{126}_{53}I$	$^{126}_{54}Xe$	1.22	$6.7\cdot10^{-3}$	$3.4\cdot10^{-3}$	0.290
$^{130}_{54}Xe$	$^{130}_{55}Cs$	$^{130}_{56}Ba$	0.250	$4.8\cdot10^{-3}$	$3.5\cdot10^{-8}$	$1.5\cdot10^{-5}$
$^{132}_{54}Xe$	$^{132}_{55}Cs$	$^{132}_{56}Ba$	1.52	$4.7\cdot10^{-3}$	$3.4\cdot10^{-3}$	0.186
$^{136}_{56}Ba$	$^{136}_{57}La$	$^{136}_{58}Ce$	0.375	$2.3\cdot10^{-3}$	$1.5\cdot10^{-7}$	$4.8\cdot10^{-5}$
$^{138}_{56}Ba$	$^{138}_{57}La$	$^{138}_{58}Ce$	3.44	$3.0\cdot10^{-3}$	0.510	1.52
$^{144}_{60}Nd$	$^{144}_{61}Pm$	$^{144}_{62}Sm$	0.188	$7.4\cdot10^{-3}$	$5.4\cdot10^{-9}$	$2.3\cdot10^{-6}$
$^{152}_{62}Sm$	$^{152}_{63}Eu$	$^{152}_{64}Gd$	0.064	$8.4\cdot10^{-4}$	$4.9\cdot10^{-4}$	0.023

$^{156}_{64}Gd$	$^{156}_{65}Tb$	$^{156}_{66}Dy$	0.086	$1.9\cdot10^{-4}$	$2.8\cdot10^{-8}$	$3.7\cdot10^{-6}$
$^{158}_{64}Gd$	$^{158}_{65}Tb$	$^{158}_{66}Dy$	0.104	$3.3\cdot10^{-4}$	0.066	0.070
$^{162}_{66}Dy$	$^{162}_{67}Ho$	$^{162}_{68}Er$	0.094	$3.1\cdot10^{-4}$	$4.0\cdot10^{-7}$	$3.8\cdot10^{-5}$
$^{164}_{66}Dy$	$^{164}_{67}Ho$	$^{164}_{68}Er$	0.104	$3.6\cdot10^{-3}$	0.042	0.040
$^{168}_{68}Er$	$^{168}_{69}Tm$	$^{168}_{70}Yb$	0.062	$2.7\cdot10^{-4}$	$6.5\cdot10^{-7}$	$2.9\cdot10^{-4}$
$^{174}_{70}Yb$	$^{174}_{71}Lu$	$^{174}_{72}Hf$	0.064	$3.1\cdot10^{-4}$	$8.6\cdot10^{-4}$	$4.8\cdot10^{-3}$
$^{180}_{72}Hf$	$^{180}_{73}Ta$	$^{180}_{74}W$	0.060	$4.0\cdot10^{-4}$	0.037	0.032
$^{184}_{74}W$	$^{180}_{75}Re$	$^{184}_{76}Os$	0.092	$1.2\cdot10^{-4}$	$1.5\cdot10^{-10}$	$3.1\cdot10^{-8}$
$^{190}_{76}Os$	$^{190}_{77}Ir$	$^{190}_{78}Pt$	0.182	$1.8\cdot10^{-4}$	$1.7\cdot10^{-8}$	$1.6\cdot10^{-6}$
$^{196}_{78}Pt$	$^{196}_{79}Au$	$^{196}_{80}Hg$	0.357	$3.1\cdot10^{-4}$	$1.0\cdot10^{-6}$	$6.7\cdot10^{-5}$

In this case even the results of calculation for the abundances of those 13 *p*-nuclei make sense in the synthesis of which the beta-decay mechanism is seemingly not dominant. These abundances, even being small, would be able to replace those zero values which are usually taken for *p*-isotopes as initial data at the pre-explosive and explosive stages of stellar evolution.

Heating the stellar matterup tothe "nuclear" temperatures is especially necessary for the first stage – overcoming the energy threshold. For its overcoming, as it follows from the results of half-life calculations for beta-stable nuclei the temperature reached by the oxygen layer burning in a massive star are sufficient. At the same time, the intensity of the beta-decay is sufficient in most cases, about which half-lives of many nuclei are evident (see Table I). These half-lives are not much different from those of natural radioactive isotopes in magnitude. If to consider a cooler but much longer stage of helium burning, beta-decay intensification of stable nuclei by thermal field of the star turns out to be too insignificant to make the *p*-nucleus yield significant.

VI. REFERENCES

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